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Consider the matrix A given by $A = \begin{bmatrix} 1 & 2 & 3 & \cdots & 100 \\ 2 & 3 & 4 & \cdots & 101 \\ 3 & 4 & 5 & \cdots & 102 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 100 & 101 & 102 & \cdots & 199 \end{bmatrix}$. Find the rank of A.

Solution: Let $A = [a_{ij}]$ where $a_{ij} = i + j - 1$ for $1 \le i, j \le 100$. Then $a_{i+1,j} - a_{ij} = 1$ for all i, j. Hence the second row minus the first row is the vector $(1, 1, \ldots, 1)$, and every subsequent row differs from the previous one by the same vector. Therefore $R_{i+1} - R_i = R_2 - R_1$ for $i = 1, 2, \ldots, 99$, so all rows lie in the one-dimensional affine subspace $R_1 + t(R_2 - R_1)$. Subtracting R_1 from each row gives a matrix whose rows are $0, (1, 1, \ldots, 1), (2, 2, \ldots, 2), \ldots, (98, 98, \ldots, 98)$, which has rank 1. Hence $\operatorname{rank} A = 1$.